1.1 Control Volume Analysis. Determine the drag of a cylinder in a very large wind tunnel (so that we can ignore wall effects). Effectively the flow is unconfined. The Reynolds number is large enough such that the wake width is reasonably steady. One approach to determine the drag is to use a force-balance. An alternative approach is to use a wake-survey. In this approach we just need to measure the velocity profile in the wake. You should assume 2-dimensional, incompressible flow with the wake velocity profile shown below:



What is the drag coefficient of the cylinder based on projected area (i.e., 2D drag coefficient based on diameter)? Is this a reasonable value (why or why not)? Solve the problem using one of the two control volumes shown above: 1) a control volume that follows streamlines (blue volume  $CV_1$ ), or 2) a rectangular control volume (red volume  $CV_2$ ).

**1.2** *CFD*. Background (optional, as much as needed):

- Complete this guided tutorial for an airfoil analysis and validation with StarCCM+. Note that step 17 should read Wing (not Domain).
- Read the best practices guide for incompressible aerodynamics in StarCCM+.
- Watch these videos on our resources page: Import 3D curve, Create 2D mesh, and Meshing.

Actual problem (required):

Perform a grid convergence study for an *inviscid* airfoil analysis. Use a NACA 2412 airfoil. Turn in a plot showing convergence in lift coefficient and drag coefficient (separate plots) as a function of some mesh parameter (e.g., base size, number of cells, etc.).

**1.3** Potential Flow. Vortex rings are pretty interesting. Read this wikipedia article to get an idea of what they are. They can propel themselves quite far when moving through a quiescent fluid. See some fun videos of vortex rings formed by a volcano, dolphins, and a plate in a pool. They can also be dangerous, for example with helicopters that descend too quickly (skip to 1:15).

If you're really careful, and put one vortex ring right behind another you can have leapfrogging vortices! Check out this video of a real-life visualization of the phenomenon, a 2D simulation, and a neat 3D simulation.

We are going to simulate the leapfrogging vortices in 2D. If you picture a toroidal vortex intersecting a 2D plane you will have two vortices (like the left pair shown below). From our understanding of vortices we can see how it is self-propelling. Consider the left pair of vortices (e.g., one ring vortex). The top vortex induces a velocity on the bottom vortex towards the right. Simultaneously, the top vortex induces a velocity on the top vortex to the right. Thus, it pushes itself forward. With two vortices next to each other, things become even more interesting.

In this example, make all vortices of equal strength ( $\Gamma$ ). Each vortex is free to move, and would move with the local fluid velocity. Plot the trajectory of each vortex. You need to use a small enough time step to ensure stability. You may need to try smaller time steps to make sure that your simulation is accurate.



If you just look at trajectories it is hard to tell that they are leapfrogging. Though not required, you might like to create an animation so that you can actually watch the movement in time. You might also like to draw a line between the vortex pairs to better visualize them as a connected ring.